

Kirchhoff's Rules

Introduction In the laboratory Simple Circuits, *Kirchhoff's second rule*, the *loop rule*, was introduced. In today's laboratory, we will review the second rule and study *Kirchhoff's first rule*, the *junction rule*. These rules represent fundamental principles that apply to any circuit. They are fundamental since, as pointed out in the textbook in Section 26-3, the first rule is an alternative statement of conservation of charge and the second rule is based on conservation of energy. In an attempt to firm up our understanding of these concepts, we will continue to study circuits that involve only resistors and sources of *emf*. As usual, however, our focus will be on the physics of the circuits.



Power Supply, PS

In the first part of this lab, we will study resistors connected in *series* and in the second part we will study resistors connected in *parallel*. Combinations of resistors are sometimes known as *resistor networks*. One of the important characteristics of a resistor network is the **equivalent resistance**, R_{eq} . R_{eq} is merely the total resistance of the network. Alternatively, R_{eq} can be thought of as the resistance of a single resistor that can replace the network. In each case, in order to study Kirchhoff's rules, we will add a source of *emf* to the resistor network then measure the voltages and currents associated with the resultant circuit. In the last part of this lab, we will study a circuit containing more than one *emf*.

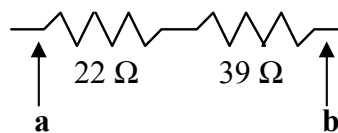
I. Resistors in Series Our first experiment is to study resistors placed one after another i.e. in *series*. As shown in section 26-2 of the textbook, the equivalent resistance of series resistors is given by

Text eq. [26-2]
$$R_{eq} = R_1 + R_2 + R_3 + \dots \quad (1)$$

1. Locate a $22\ \Omega$ and a $39\ \Omega$ resistor. Measure the resistances with a multimeter and record the values in the space provided.

R1: $R_{22\Omega} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}\ \Omega$ **R2:** $R_{39\Omega} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}\ \Omega$

2. Connect the resistors in series as shown at the right.



3. Use eq. (1) to predict the equivalent resistance of the combination of resistors and record the value in the space provided.

R3: $R_{eq,predicted} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}\ \Omega$ **R4:** $R_{eq,measured} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}\ \Omega$

4. Use a multimeter to measure the total resistance of the combination of resistors (The total resistance is the resistance between points **a** and **b**.) and record the value in the space provided.

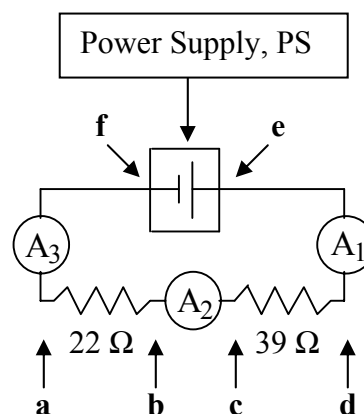
R5: Compare the predicted and measured values of R_{eq} .

5. Be sure that a DCA is connected to CH 1 and CH 2 on the LabPro. Check that a voltage probe is connected as PROBE 1 and a current probe is connected as PROBE 2. Finally, be sure that a second DCA is connected to CH 3 and CH 4 on the LabPro and that two current probes are connected to the second DCA.

6. Add a source of *emf* (the Hewlett Packard 6214C Power Supply, PS, shown on the front page of this write-up) and current probes (ammeters) to the series resistors to construct the circuit shown at the right.

Note: The two terminals at the lower right of the PS (red tipped terminal marked with a minus sign – and black tipped terminal marked with the following symbol for ground \perp) *must be connected together* (shorted).

Note: The black terminal on the side of each DCA *must be connected to the ground on the PS.*



7. Start the program **LoggerPro**. When the program starts, open the folder **Kirch Rules** and open the file **Volt3Cur**.

8. When the program is ready, short out (connect together) the voltage probe leads. Also, be sure that there is no current through the current probes. (All that is necessary is to disconnect one wire in the circuit.) **Zero** all probes.

9. Connect the voltmeter so that it will read the voltage $V_{\text{ef}} = V_{\text{e}} - V_{\text{f}}$ (red lead at point **e** and black lead at point **f**). Set the PS to read about 3 V. It may be necessary to turn up both the CURRENT and VOLTAGE knobs on the PS in order to obtain 3 V. (Note: *The voltage readings appear at the bottom of the computer screen so that it is not necessary to click on **Collect** to obtain voltage readings.*)

10. Click on **Collect** to obtain a display of the voltage and currents.

R6: Depending on the instructions from your instructor, either **Print** the graphs or **Copy** the graphs and **Paste** them into an **Excel** spreadsheet so that the upper left corner of the first one is in cell A1. If you are using **Excel**, resize the graphs so that the right edges do not extend beyond column J.

11. Remembering that the current probes have an absolute accuracy (calibration) of 2%, record the experimental values of I_1 , I_2 and I_3 in the space provided.

R7: $I_1 = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \text{ A}$

R8: $I_2 = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \text{ A}$

R9: $I_3 =$ _____ \pm _____ A

R10: Do the data show that $I_1 = I_2 = I_3$? Give a “physics” reason that it must be true that $I_1 = I_2 = I_3$. (It is not sufficient to merely say that the current must be the same everywhere in a series circuit, although that is true.)

12. Calculate the average value of I_1 , I_2 and I_3 and record the value in the space provided. We will use this as our best value of the current in the series circuit.

R11: $I_{\text{ave}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ A}$

13. Use the voltage probe to measure the voltages indicated below. We will assume that the voltage probe has an absolute accuracy of about 1%.

R12: $V_{\text{ef}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$ **R13:** $V_{\text{de}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$

R14: $V_{\text{cd}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$ **R15:** $V_{\text{bc}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$

R16: $V_{\text{ab}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$ **R17:** $V_{\text{fa}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$

The measurements should show that V_{ef} is positive and all the other voltages are negative (or zero). A voltage is negative if there is a decrease in potential across the resistor in the “direction” of measurement (from the black lead to the red lead). Unfortunately, on p. 665, the textbook uses the term *voltage drop* to describe a negative voltage i.e. the term *voltage drop* is used interchangeably with the correct term *potential drop*. Consequently, there will be no questions on the final exam that require a distinction between the terms voltage drop and potential drop.

We can now test Kirchhoff's second rule, the loop rule. This is stated in the textbook (p. 665) as **“the sum of the changes in potential around any closed path of a circuit must be zero.”** Consequently, the loop rule requires that the sum of the voltages be zero.

14. Calculate the sum of the voltages and record the value in the space provided.

R18: Sum of $V_s = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$

R19: Do the results confirm the loop rule? Discuss.

15. Use the value of each resistance, the average current and $V_{\text{xy}} = IR$ to predict the value of the voltage across each resistor. Record your predictions in the space provided.

R20: $V_{22\Omega, \text{predicted}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$

R21: $V_{39\Omega, \text{predicted}} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$

R22: Are the predicted and measured values the same to within the experimental uncertainty? (Be sure to specify which of the six voltages from R12 to R17 that you are comparing with $V_{22\Omega, \text{predicted}}$ and $V_{39\Omega, \text{predicted}}$.)

R23: Is the voltage across the $22\ \Omega$ resistor equal to the voltage across the $39\ \Omega$ resistor? Explain.

R24: Compare V_{ef} and the sum of the voltages across the resistors. Explain why V_{ef} and the sum of the voltages across the resistors should be opposite in sign and approximately equal in magnitude.

16. Finally, use the measured voltages, the measured current and $V_{xy} = IR$ to calculate the equivalent resistance of the series resistors, $R_{\text{eq, calculated}}$. Record the value of $R_{\text{eq, calculated}}$ in the space provided.

R25: $R_{\text{eq, calculated}} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}\ \Omega$

R26: Which voltage or voltages did you use in the calculation of $R_{\text{eq, calculated}}$? Show your calculation.

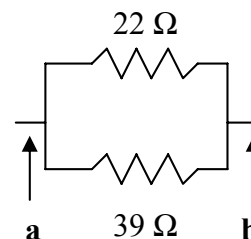
R27: Compare the value of $R_{\text{eq, calculated}}$ with the values of $R_{\text{eq, predicted}}$ and $R_{\text{eq, measured}}$ recorded in steps 3 and 4.

II. Resistors in Parallel We now branch out to consider resistors that are placed side by side i.e. in *parallel*. As shown in the textbook in section 26-2, the equivalent resistance of parallel resistors is given by

Text eq. [26-3]
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \quad (2)$$

1. Connect the $22\ \Omega$ and $39\ \Omega$ resistors in parallel as shown at the right.

2. Use eq. (2) to predict the equivalent resistance of the combination of



resistors and record the value in the space provided.

R28: $R_{\text{eq,predicted}} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \Omega$

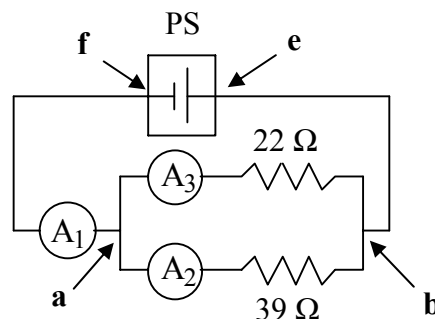
R29: $R_{\text{eq,measured}} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \Omega$

3. Use a multimeter to measure the total resistance of the combination of resistors (The total resistance is the resistance between points **a** and **b**.) and record the value in the space provided.

R30: Compare the predicted and measured values of R_{eq} .

We now have an opportunity to study Kirchhoff's first rule, the junction rule. This is stated in the textbook (p. 665) as "**at any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.**" We will define a junction point to be the point of intersection of three or more wires.

This (the junction rule) was of no use in the study of a series circuit since there was no junction. However, it should be apparent that there are junctions at points **a** and **b** for the parallel resistors shown in the previous diagram and in the diagram at the right. Let us study the currents at point **a**.



4. Add an *emf* (PS) and ammeters to the parallel resistors to construct the circuit diagram shown.

5. Connect the voltage probe so that it reads the voltage $V_{\text{ef}} = V_e - V_f$ (red lead at point **e** and black lead at point **f**) and set the PS so that it reads about 3 V.

6. Click on **Collect** to obtain a display of the voltage and currents.

R31: Depending on the instructions from your instructor, either **Print** the graphs or **Copy** the graphs and **Paste** them into an **Excel** spreadsheet so that the upper left corner of the first one is in cell K1. If you are using **Excel**, resize the graphs so that the right edges do not extend beyond column T.

7. Record the values of the currents in the space provided.

R32: $I_1 = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \text{ A}$

R33: $I_2 = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \text{ A}$

R34: $I_3 = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \text{ A}$

R35: Do the data verify the junction rule? Explain. (Be sure to write the expected relationship between I_1 , I_2 and I_3 .)

8. Use the voltage probe to measure the voltage across each resistor. Record the values in the space provided.

R36: $V_{39\Omega} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$ **R37:** $V_{22\Omega} = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \text{ V}$

The data should show that $V_{39\Omega} \approx V_{22\Omega}$, however, we know that the voltage must be the same across each resistor when they are in parallel. **R38:** What is the reason that these voltages should only be approximately equal? (Hint: Note the positions of the ammeters.)

We could apply the loop rule to this circuit (the circuit containing parallel resistors) since there are three independent loops. However, we won't bother with that now since we will do that for a "multi-loop circuit" in Part III of this lab. Before we go on to Part III, however, let us estimate the value of R_{eq} from our data. Consider the following equation

$$R_{\text{eq}} \approx \frac{V_{\text{ef}}}{I_1} \quad (3)$$

R39: Discuss eq. (3). Be sure to include why V_{ef} and I_1 are used and why the equation is only approximate.

9. Calculate the value of R_{eq} using the data and eq. (3) and record the value in the space provided.

R40: $R_{\text{eq,calculated}} \approx \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}} \Omega$

R41: Compare the value of $R_{\text{eq,calculated}}$ with the values of $R_{\text{eq,predicted}}$ and $R_{\text{eq,measured}}$ recorded in steps 2 and 3 of this section.

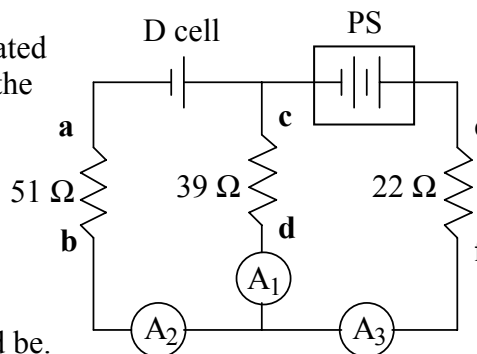
III. A More Complicated Circuit Finally, we turn our attention to a circuit composed of more than one loop and at least one *emf* in at least two of the loops. This is sometimes known as a "multi-loop circuit" though the term does not appear in our textbook. What we will do is predict what the currents should be assuming that the voltages and resistances are known. We will then compare the predicted currents with the measured values.

1. Construct the circuit shown in the next diagram. Use a D cell battery on the left and the DC power supply on the right. Set the PS for about 3 V.

2. Use the voltage probe to measure the voltages associated with the battery and power supply, V_{ca} and V_{ec} . Record the values in the space provided.

R42: $V_{ca} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \text{ V}$

R43: $V_{ec} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} \text{ V}$



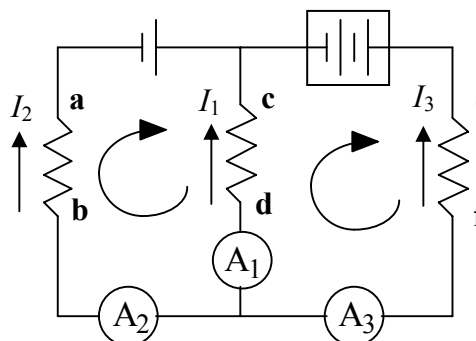
Let us now predict what the currents, I_1 , I_2 and I_3 , should be.

Following the procedure outlined on pages 666 and 667 of the textbook, we will assume directions for the currents, I_1 , I_2 and I_3 . For convenience, let us assume that the directions of all of the currents are “up” through the resistors as shown in the next diagram. Of course, they can’t all be “up” so we know that the value of at least one of the currents must be negative.

R44: Why can’t all of the currents be “up?”

R45: Use the junction rule to write an equation relating I_1 , I_2 and I_3 . (Yes, this was the last question on the Pre-Lab Exercise.)

R46: Use the loop rule to write two more equations that relate I_1 , I_2 and I_3 . Use the loop on the left of the circuit and the loop on the right of the circuit and travel around each loop in the direction shown. (Ignore the resistance of the ammeters since the resistance of an ammeter is small.)



3. Solve these three equations and three unknowns to predict what the currents should be.

R47: Discuss how you solved the equations. Record the results in the space provided.

R48: $I_{1,\text{predicted}} = I_{39\Omega,\text{predicted}} = \underline{\hspace{2cm}}$ A up or down?

R49: $I_{2,\text{predicted}} = I_{51\Omega,\text{predicted}} = \underline{\hspace{2cm}}$ A up or down?

R50: $I_{3,\text{predicted}} = I_{22\Omega,\text{predicted}} = \underline{\hspace{2cm}}$ A up or down?

4. To make the interpretation of the data easier, make the arrow on each current probe (ammeter) point “up” toward the adjacent resistor i.e. make the arrow on A_1 point toward the 39 Ω resistor, the arrow on A_2 point toward the 51 Ω resistor and the arrow on A_3 point toward the 22 Ω resistor. This is the experimental equivalent of choosing all of the currents “up.”

5. Click on **Collect** to measure the currents.

R51: Depending on the instructions from your instructor, either **Print** the graphs or **Copy** the graphs and **Paste** them into an **Excel** spreadsheet so that the upper left corner of the first one is in cell U1. If you are using **Excel**, resize the graphs so that the right edges do not extend beyond column AD.

R52: $I_{1,\text{measured}} = I_{39\Omega,\text{measured}} = \underline{\hspace{2cm}}$ A up or down?

R53: $I_{2,\text{measured}} = I_{51\Omega,\text{measured}} = \underline{\hspace{2cm}}$ A up or down?

R54: $I_{3,\text{measured}} = I_{22\Omega,\text{measured}} = \underline{\hspace{2cm}}$ A up or down?

R55: Compare the predicted and measured values of the currents.

End of Lab Checkout Before leaving the laboratory, please dismantle any circuits or connections that you have made. Place the wires in one pile and return the meters to their boxes. Show your instructor that your meters work properly.